

# A study of the pion effective mass at finite temperature using the linear sigma model <sup>\*</sup>

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## Abstract

We have used the Cornwall–Jackiw–Tomboulis method of composite operators and its formulation at finite temperature by Amelino-Camelia and Pi, in order to calculate the effective potential of the  $O(4)$  linear sigma model beyond the Hartree approximation. We have obtained a system of gap equations for the effective mass of sigma and the pions as well as for the condensate, the order parameter of the chiral phase transition. We find that the thermal effective mass of the pions at low temperatures remains lower than in the Hartree case, nevertheless deviates from the chiral limit. Our observation is consistent with other results which have been published previously.

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The linear sigma model is very often used as an effective theory to QCD in order to obtain insight into the nature of the chiral phase transition. Among the quantities of interest is the velocity of pions in medium as well as the pion dispersion relation and pion dissipative properties. This is a subject which has attracted some attention and has been studied in a variety of models, including the Skyrme model [1], the linear sigma model [2, 3, 4, 5] as well as chiral perturbation theory [6, 7, 8, 9].

The essential physics is included in the energy expression of the pions propagating in a thermal bath

$$p_0^2 = \mathbf{p}^2 + m_\pi^2 + \Sigma_\pi(p_0, \mathbf{p}) , \quad (1)$$

where  $\Sigma_\pi(p_0, \mathbf{p})$  is the pion self-energy and depends strongly on the physical conditions of the medium in which the pion propagates. When the medium is in thermal equilibrium, the self-energy is determined by the temperature. The self-energy contains a real part which is related to pion velocity and dispersion, while the imaginary part encodes the information about the pion absorption in the medium where it propagates.

We use the  $O(4)$  version of the linear sigma model with the Lagrangian given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}m^2\Phi^2 - \frac{1}{4!}\lambda(\Phi^2)^2 - \varepsilon\sigma , \quad (2)$$

where  $\Phi = (\sigma, \pi_1, \pi_2, \pi_3)$ , while for  $\varepsilon \neq 0$  we deviate from the chiral limit and the pions are massive. In order to study the propagation properties of the pions at finite temperature, we have obtained an expression for the thermal effective potential of the linear sigma model up to two loops [10]. As it is shown in Fig. 1, there are two types of graphs which contribute to the effective potential up to this level.

We have used the method of composite operators which is a nice way to perform systematic selective summations in the loop expansion of the effective action and the effective potential. In this case, the effective action  $\Gamma(\phi, G)$  is the generating functional of the two-particle irreducible (2PI) vacuum graphs and depends on the constant background field  $\phi(x)$ , as well as on the dressed propagators  $G(x, y)$ . This formalism was initially used by Cornwall-Jackiw-Tomboulis (CJT) [11] for a study of the  $O(N)$  model at zero temperature, but it has been generalized to finite temperature by Amelino-Camelia and Pi [12], and was used for investigations of the effective potential of the  $\lambda\phi^4$  theory, the linear sigma model and gauge theories. The

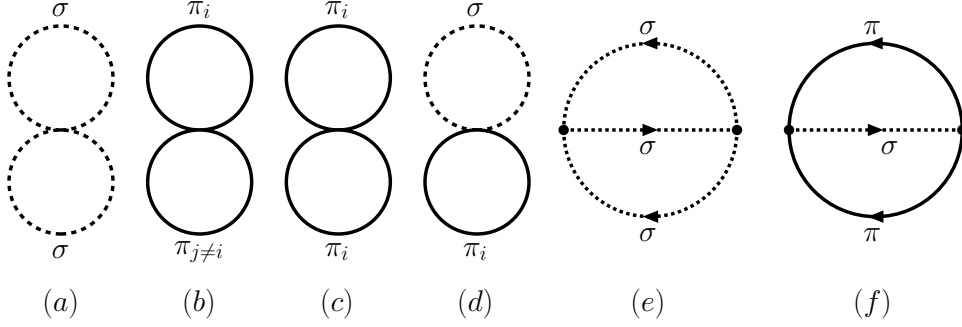


Figure 1: The set of 2PI graphs which contribute to the thermal effective potential of the O(4) linear sigma model up to two loops. The graphs a-d are usually called double-bubble while e,f sunset diagrams.

method of composite operators was used long ago in condensed matter context by Luttinger and Ward [13], as well as by Baym [14] in self-consistent approximations of many body systems. Nowadays variations of the technique are very popular and it has many applications in systems at finite temperature in or out of equilibrium as discussed in detail in the contributions of Berges and Blaizot in these proceedings.

According to CJT method, the functional derivatives of the effective potential with respect to dressed propagators and the background field result in a system of gap equations for the thermal effective masses of pions and sigma as well as for the condensate. We have examined the problem in two limiting cases. First in the Hartree approximation where we have taken into account only the graphs (a)-(d) in Fig. 1. In this case the self-energy only contains a real part. Going beyond Hartree and guided by the low energy theorem we have performed a selective summation of the relevant graphs so we have considered only the graph (f) in Fig. 1. Eventually we have ended up with a gap equation for the thermal effective mass of the pions valid only in the low temperature region. For this calculation we have made the assumption that the sigma mass varies little with temperature. We were able this way to obtain the contribution to the thermal pion mass due to the real part of the pion self energy. Details of this study have been presented in [10], while the Hartree case is discussed in [15] as well as by Rischke and Lenaghan [16].

Our conclusion is reflected in Fig. 2. There we can see that even if we have started with massless pions ( $\varepsilon = 0$ ), we find that in the Hartree approx-

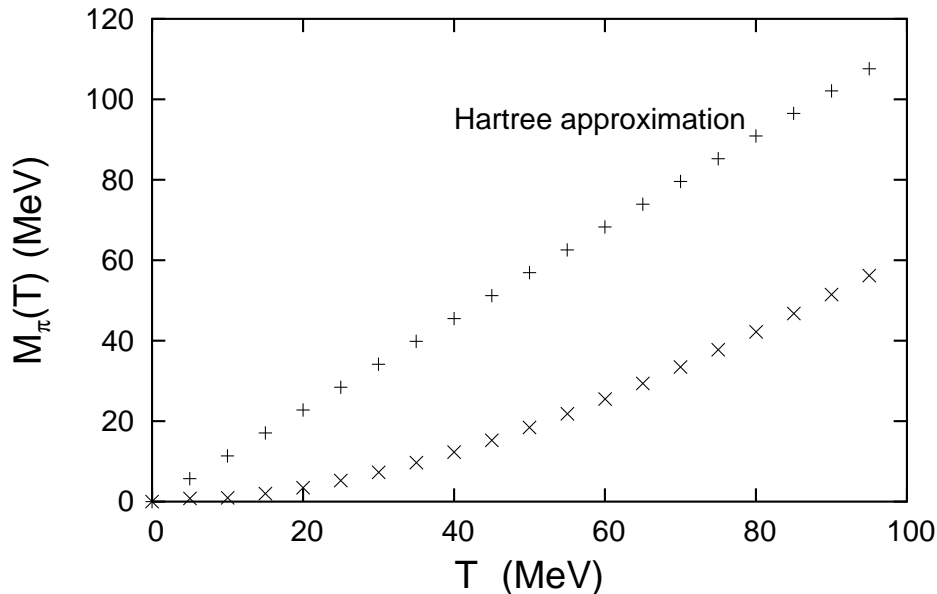


Figure 2: The thermal effective pion mass as a function of temperature.

imation, the thermal pion mass grows linearly with temperature [15, 16]. On the contrary, taking into account the sunset diagram (Fig. 1f), we find that the thermal mass depends on temperature quadratically [10]. This of course, at low temperature, keeps the pions closer to their Goldstone nature. Our observation is consistent with earlier results by Itoyama–Mueller [17] and Pisarski–Tytgat [4] using the linear sigma model, as well as with other investigations using the techniques of chiral perturbation theory [18].

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